Directional derivatives.

$\boxed{D_u(f(x,y))} = \nabla f \cdot u$

$= \langle f_x, f_y \rangle \cdot \langle a, b \rangle$, $u = \langle a, b \rangle$

$= |\nabla f| \cdot |u| \cos\theta$

$= |\nabla f| \cos\theta \to$ max $|\nabla f|$, $\theta = 0°$

min $-|\nabla f|$, $\theta = 180°$

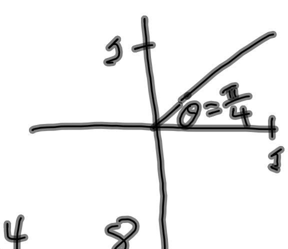
Level (plane) $D_u(f) = 0$

$\theta = 90°$

4–6 Find the directional derivative of f at the given point in the direction indicated by the angle θ.

$u = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

4. $f(x, y) = x^2y^3 - y^4$, $(2, 1)$, $\theta = \pi/4$



$f_x = 2xy^3 = \underline{4}$

$f_y = 3x^2y^2 - 4y^3 = 12 - 4 = \underline{8}$

$\langle 4, 8 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle = \frac{4}{\sqrt{2}} + \frac{8}{\sqrt{2}}$

14. $g(r, s) = \tan^{-1}(rs)$, $(1, 2)$, $\mathbf{v} = \underbrace{5\mathbf{i} + 10\mathbf{j}}$

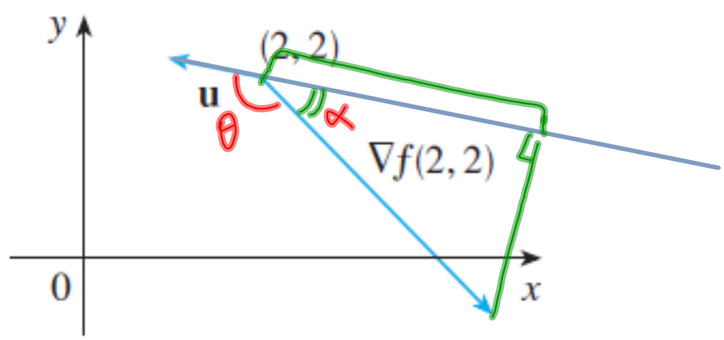
$$g_r = \frac{1}{1+r^2s^2}\cdot s \qquad \frac{\langle 5,10\rangle}{\sqrt{125}} = \left\langle \frac{1}{\sqrt5}, \frac{2}{\sqrt5}\right\rangle$$

$$g_s = \frac{1}{1+r^2s^2}\cdot r$$

$$D_u\big(g(r,s)\big) = \left\langle \frac{2}{5}, \frac{1}{5}\right\rangle \cdot \left\langle \frac{1}{\sqrt5}, \frac{2}{\sqrt5}\right\rangle$$

$$= \frac{2}{5\sqrt5} + \frac{2}{5\sqrt5} = \frac{4}{5\sqrt5}$$

18. Use the figure to estimate $D_{\mathbf{u}} f(2, 2)$. ≈ -1.7



$$\nabla f \cdot u$$

$$= |\nabla f| \cdot \cos\theta = -|\nabla f| \cos\alpha$$